**FIRST MOVER ADVANTAGE**

SYMMETRICAL, TWO PERSON GAME OF CHANCE

(e.g. a coin to be tossed, first player landing H wins)

F: first player; S: second player.

B: probability that F or S wins on a given turn (= 0.5 for a fair coin)

X: probability that F (that is, first mover) wins eventually.

ONE APPROACH:

F wins the game

if and only if

F wins on first turn OR S **loses** the game after the first turn.

🡪 X = B + (1-B)(1-X) ... application of sum rule, product rule

= B + 1 - B - X + BX

= 1 - X + BX

🡪 (2-B)X = 1

🡪 X = 1/(2-B)

SECOND APPROACH:

F wins the game

if and only if

F wins on first turn OR (S loses on second turn & F wins eventually).

🡪 X = B + (1-B)(1-B)X ... application of sum rule, product rule

= B + X - 2BX + B2X

Cancel X from LHS and RHS; then divide by B, assuming it is not zero:

🡪 0 = 1 -2X + BX

🡪 X = 1/(2-B)



ANOTHER VARIATION

Suppose **three** players – F, S and T – toss a coin by turns. The first one to land H is the winner. What is the probability that F, S or T wins?

[Note: The coin may or may not be biased.]

Following the second approach above, assuming B ~= 0:

🡪 X = B + (1-B)3X

Solving as before:

X = 1/(B2 - 3B + 3)

A fair coin, with B = ½, gives: X = 4/7. [Exercise: Find the probabilities of S and T winning with fair coin.]